S 25 Nr 15

a) $f(x) = \frac{1}{2}x^2$; $B(1 \mid f(1))$ Berührpunkt

$B(x_0 \;\; f(x_0))$

$f'(x) = \frac{1}{2} \cdot 2x \Rightarrow f'(1) = \frac{1}{2} \cdot 2 \cdot 1 = 1 = f'(x_0)$

$\boxed{t(x) = f'(x_0)(x - x_0) + f(x_0)}$

$t(x) = 1 \cdot (x - 1) + \frac{1}{2} 1^2 = x - 1 + \frac{1}{2} = x - \frac{1}{2}$ Tangentengleichung

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b) $f(x) = 2x^2 - 4$ $B(-2 \mid f(-2))$

$f'(x) = 2 \cdot 2x \Rightarrow f'(x_0) = f'(-2) = 2 \cdot 2\,(-2) = -8$

$t(x) = \underbrace{-8}_{f'(x_0)}(x - \underbrace{(-2)}_{x_0}) + \underbrace{2 \cdot (-2)^2 - 4}_{f(x_0)} = -8x - 16 + 8 - 4 =$

$t(x) = -8x - 12$ Tangentengleichung

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c) $f(x) = \sqrt{x}$, $B(0{,}5 \mid f(0{,}5))$

$f'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \Rightarrow f'(x_0) = f'(0{,}5) = \frac{1}{2} \cdot \frac{1}{\sqrt{0{,}5}} = \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{1}{2}}} = \frac{1}{2} \cdot \sqrt{2}$

$t(x) = \underbrace{\frac{1}{2} \cdot \sqrt{2}}_{f'(x_0)}(x - \underbrace{\frac{1}{2}}_{x_0}) + \underbrace{\sqrt{\frac{1}{2}}}_{f(x_0)} = \frac{1}{2} \cdot \sqrt{2} \cdot x - \frac{1}{4}\sqrt{2} + \sqrt{\frac{1}{2}}$

$t(x) = \frac{1}{2}\sqrt{2} \cdot x - \frac{1}{4}\sqrt{2} + \sqrt{\frac{1}{2} \cdot \frac{2}{2}} = \frac{1}{2}\sqrt{2}\,x - \frac{1}{4}\sqrt{2} + \frac{1}{2}\sqrt{2}$

$t(x) = \frac{1}{2}\sqrt{2} \cdot x - \frac{1}{4}\sqrt{2} \approx 0{,}707 \cdot x - 0{,}354$

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d) $f(x) = -x^3 + 2$; $B(2 \mid f(2))$

$f'(x) = -1 \cdot 3 \cdot x^2 \Rightarrow f'(x_0) = f'(2) = -1 \cdot 3 \cdot 2^2 = -12$

$t(x) = \underbrace{-12}_{f'(x_0)} \cdot (x - \underbrace{2}_{x_0}) + \underbrace{(-2^3 + 2)}_{f(x_0)} = -12x + 24 - 8 + 2$

$t(x) = -12x + 18$