S 24 Nr. 5

a) $f(x) = 3x + 2$, $x_0 = 4$, $x_1 = 9$

$$m_{x_0}(h) = \frac{3(x_0+h)+2-\{3x_0+2\}}{h} = \frac{\cancel{3x_0}+3h\cancel{+2}-\cancel{3x_0}\cancel{-2}}{h} = \frac{3h}{h} = 3$$

$$\lim_{h\to 0} m_{x_0}(h) = \lim_{h\to 0}(3) = \underline{\underline{3 = f'(x_0)}} \Rightarrow \underline{\underline{f'(4) = 3}} \; ; \; \underline{\underline{f'(9) = 3}}$$

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b) $g(x) = mx + c$

$$M_{x_0}(h) = \frac{g(x_0+h)-g(x_0)}{h} = \frac{m(x_0+h)+c-\{mx_0+c\}}{h}$$

$$M_{x_0}(h) = \frac{\cancel{mx_0}+mh\cancel{+c}-\cancel{mx_0}\cancel{-c}}{h} = \frac{m\cdot h}{h} = m$$

$$\lim_{h\to 0} M_{x_0}(h) = \lim_{h\to 0} m = \underline{\underline{m = f'(x_0)}}$$

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c) Die Ableitung einer linearen Funktion ist an jeder Stelle des Definitionsbereichs konstant $m = f'(x_0)$. Sie entspricht der Steigung der Geraden.

S 24 Nr. 6

$f(x) = x^3$; $x_0 = 1$

$$m_{x_0}(h) = \frac{(x_0+h)^3 - x_0^3}{h} = \frac{(x_0+h)\cdot(x_0+h)^2 - x_0^3}{h}$$

$$m_{x_0}(h) = \frac{(x_0+h)(x_0^2+2x_0h+h^2)-x_0^3}{h} =$$

$$m_{x_0}(h) = \frac{\cancel{x_0^3}+2x_0^2h+x_0h^2+x_0^2h+2x_0h^2+h^3-\cancel{x_0^3}}{h} = \frac{3x_0^2h+3x_0h^2+h^3}{h}$$

$$m_{x_0}(h) = \frac{\cancel{h}\cdot(3x_0^2+3x_0h+h^2)}{\cancel{h}} = 3x_0^2+3x_0h+h^2$$

$$\lim_{h\to 0} m_{x_0}(h) = \lim_{h\to 0}(3x_0^2+\underbrace{3x_0h}_{\to 0}+\underbrace{h^2}_{\to 0}) = \underline{\underline{3x_0^2 = f'(x_0)}} \Rightarrow f'(1) = 3\cdot 1^2 = 3$$

für $h \to 0$