S 65 Nr 4

b) $f_a(x) = x^2 - ax - 1 \quad ; \quad a > 0$

Nullstellen. $f_a(x) = 0$

$$x^2 - ax - 1 = 0 \Rightarrow x_{1,2} = +\frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 + 1}$$

$$\underline{\underline{N_1\left(+\frac{a}{2} - \sqrt{\left(\frac{a}{2}\right)^2+1} \,\middle|\, 0\right)}} \vee \underline{\underline{N_2\left(+\frac{a}{2} + \sqrt{\left(\frac{a}{2}\right)^2+1} \,\middle|\, 0\right)}}$$

Extrema notw Bed. $f_a'(x) = 0$

$$f_a'(x) = 2x - a = 0 \Rightarrow \underline{\underline{x_3 = \frac{a}{2}}}$$

hinr. Bed: $f_a'(x) < 0$ für $x < \frac{a}{2}$ weil $f_a'(x)$ eine streng monoton wachsende Gerade ist!

$f_a'(x) > 0$ für $\frac{a}{2} < x$

$$\Rightarrow T\left(\frac{a}{2} \,\middle|\, f_a\left(\frac{a}{2}\right)\right) = \left(\frac{a}{2} \,\middle|\, \left(\frac{a}{2}\right)^2 - a\cdot\frac{a}{2} - 1\right) = \left(\frac{a}{2} \,\middle|\, \frac{a^2}{4} - \frac{a^2}{2} - 1\right) = \underline{\underline{\left(\frac{a}{2} \,\middle|\, -\frac{a^2}{4} - 1\right)}}$$

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c) $f_a(x) = a^2 \cdot x^4 - x^2$

Nullstellen: $f_a(x) = 0$

$$a^2 \cdot x^4 - x^2 = x^2 \cdot (ax^2 - 1) = 0 \Rightarrow x_1 = 0 \vee ax^2 - 1 = 0$$

$$ax^2 = 1$$

$$x_{2,3} = \pm\sqrt{\frac{1}{a}}$$

$$\underline{\underline{N_1\left(-\sqrt{\frac{1}{a}} \,\middle|\, 0\right)}} \vee \underline{\underline{N_2(0|0)}} \vee \underline{\underline{N_3\left(+\sqrt{\frac{1}{a}} \,\middle|\, 0\right)}}$$

Extrema. notw Bed. $f_a'(x) = 4a^2 \cdot x^3 - 2x = 0$

$$\Rightarrow x \cdot (4a^2 \cdot x^2 - 2) = 0 \Rightarrow x_4 = 0 = x_1 \vee 4a^2x^2 - 2 = 0 \quad | +2$$

$$4a^2x^2 = 2 \quad | : 4a^2$$

$$x^2 = \frac{2}{4a^2} = \frac{1}{2a^2}$$

$$x_{5,6} = \pm\sqrt{\frac{1}{2a^2}} = \frac{1}{a \cdot 2}\sqrt{2}$$

hinr Bed:

$f_a'(x) < 0$ für $x < -\frac{\sqrt{2}}{2a}$; $f_a'(x) > 0$ für $-\frac{\sqrt{2}}{2a} < x < 0$

$f_a'(x) < 0$ für $0 < x < +\frac{\sqrt{2}}{2a}$; $f_a'(x) > 0$ für $+\frac{\sqrt{2}}{2a} < x$

$$T_1\left(-\frac{\sqrt{2}}{2a} \,\middle|\, -\frac{1}{4a^2}\right) ; H(0|0) ; T_2\left(+\frac{\sqrt{2}}{2a} \,\middle|\, -\frac{1}{4a^2}\right)$$