S 63 Nr. 1

a) $f(x) = -100 \cdot x^2 + 0{,}01 \cdot x^5$

$f(100\,000) = f(10^5) = -10^2 \cdot (10^5)^2 + 10^{-2} \cdot (10^5)^5 = \underbrace{-10^{12}}_{<0} + \underbrace{10^{23}}_{>0}$

$|-10^{12}| < |10^{23}| \Rightarrow \underline{\underline{f(10^5) > 0}}$

$f(-100\,000) = \underline{\underline{f(-10^5)}} = \underbrace{-10^2 \cdot (-10^5)^2}_{<0} + \underbrace{10^{-2} \cdot (-10^5)^5}_{<0} \underline{\underline{< 0}}$

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b) $f(x) = x^2 - \frac{3}{x}$

$f(10^5) = (10^5)^2 - \frac{3}{10^5} = 10^{10} - \frac{3}{10^5} \Rightarrow |10^{10}| \gg |-\frac{3}{10^5}| \Rightarrow$

$\underline{\underline{f(10^5) > 0}}$

$\underline{\underline{f(-10^5)}} = (-10^5)^2 - \frac{3}{-10^5} = 10^{10} + \frac{3}{10^5} \underline{\underline{> 0}}$

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c) $f(x) = x^3 - 0{,}25x^4$

$f(10^5) = (10^5)^3 - 0{,}25 \cdot (10^5)^4 = 10^{15} - \frac{1}{4} \cdot 10^{20} < 0$

$f(-10^5) = (-10^5)^3 - 0{,}25(-10^5)^4 = -10^{15} - \frac{1}{4} \cdot 10^{20} < 0$

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d) $f(x) = 250 - x^3$

$\underline{\underline{f(10^5)}} = 250 - (10^5)^3 = 250 - 10^{15} \underline{\underline{< 0}}$

$\underline{\underline{f(-10^5)}} = 250 - (-10^5)^3 = 250 + 10^{15} \underline{\underline{> 0}}$

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e) $f(x) = -\frac{3}{x^{10}} + x$; $\underline{\underline{f(10^5)}} = -\frac{3}{(10^5)^{10}} + 10^5 \underline{\underline{> 0}}$

$\underline{\underline{f(-10^5)}} = -\frac{3}{(-10^5)^{10}} + (-10^5) \underline{\underline{< 0}}$

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f) $f(x) = x^4 - \frac{1}{x^4}$

$\underline{\underline{f(10^5)}} = (10^5)^4 - \frac{1}{(10^5)^4} = 10^{20} - \frac{1}{10^{20}} \underline{\underline{> 0}}$

$\underline{\underline{f(-10^5)}} = (-10^5)^4 - \frac{1}{(-10^5)^4} = +10^{20} - \frac{1}{10^{20}} \underline{\underline{> 0}}$