

548 Nr 4

$$a) f(x) = (x-3)(x^3-8x) = 0 \Rightarrow x-3=0 \vee x^3-8x = x(x^2-8) = 0$$

$$\underline{x_1=3} \vee \underline{x_2=0} \vee \underline{x_{3,4} = \pm\sqrt{8}}$$

$$x_{3,4} = \pm 2\sqrt{2}$$

$$\text{Nullstellen} = \{-2\sqrt{2}, 0, +2\sqrt{2}, 3\}$$

$$b) f(x) = x^3 + 2x^2 - 8x = 0 \Rightarrow x \cdot (x^2 + 2x - 8) = 0$$

$$\Rightarrow \underline{x_1=0} \vee x^2 + 2x - 8 = 0 \Rightarrow x_{2,3} = -1 \pm \sqrt{1^2 + 8} = -1 \pm 3$$

$$\text{Nullstellen} = \{+2, 0, -4\}$$

$$c) f(x) = x^4 + 4x^3 + 3x^2 = 0 \Rightarrow x^2 \cdot (x^2 + 4x + 3) = 0$$

$$\Rightarrow \underline{x_1=0} \vee x^2 + 4x + 3 = 0 \Rightarrow x_{2,3} = -2 \pm \sqrt{4-3} = -2 \pm 1$$

$$\underline{x_2=-1} \vee \underline{x_3=-3}$$

$$\text{Nullstellen} = \{-3, -1, 0\}$$

$$d) f(x) = (x^4 - 8x^2 + 16)(x^2 - 5x) = 0$$

$$\Rightarrow x^2 - 5x = 0 \Rightarrow x(x-5) = 0 \Rightarrow \underline{x_1=0} \vee \underline{x_2=5}$$

$$\vee x^4 - 8x^2 + 16 = 0 \mid \text{Sub. } x^2 = u \Rightarrow u^2 - 8u + 16 = 0$$

$$u_{1,2} = 4 \pm \sqrt{16-16} = 4$$

$$\text{Rück. Sub. } x^2 = 4 \Rightarrow \underline{x_{3,4} = \pm 2} \quad \text{Nullstellen} = \{-2, 0, +2, 5\}$$

$$e) f(x) = x^5 - 41x^3 + 400x = 0 \Rightarrow x(x^4 - 41x^2 + 400) = 0 \Rightarrow \underline{x_1=0}$$

$$\vee x^4 - 41x^2 + 400 = 0 \mid \text{Sub. } x^2 = u$$

$$u_{1,2} = \frac{41}{2} \pm \sqrt{\left(\frac{41}{2}\right)^2 - 400} = \frac{41}{2} \pm \frac{9}{2} \Rightarrow u_1 = 25 \vee u_2 = 16$$

$$\text{Rück Sub } x^2 = 25 \quad \vee \quad x^2 = 16$$

$$\underline{x_{2,3} = \pm 5} \quad \underline{x_{4,5} = \pm 4} \quad \text{Nullstellen} = \{-5, -4, 0, +4, 5\}$$

$$f) f(x) = 2x^6 - 32x^4 + 128x^2 = 0 \Rightarrow 2x^2 \cdot (x^4 - 16x^2 + 64) = 0 \Rightarrow \underline{x_1=0}$$

$$x^4 - 16x^2 + 64 = 0 \mid \text{Sub } x^2 = u \Rightarrow u^2 - 16u + 64 = 0$$

$$u_{1,2} = 8 \pm \sqrt{64-64} = 8 \Rightarrow \text{Rück Sub. } x^2 = 8 \Rightarrow x_{2,3} = \pm\sqrt{8} =$$

$$\underline{x_{2,3} = \pm\sqrt{4 \cdot 2} = \pm 2\sqrt{2}}$$

$$\text{Nullstellen} = \{-2\sqrt{2}, 0, +2\sqrt{2}\}$$