

a)  $x^5 - 20x^3 + 64x = 0$

$x(x^4 - 20x^2 + 64) = 0 \Rightarrow$

$x_1 = 0$   $\vee x^4 - 20x^2 + 64 = 0 \quad | \text{Sub. } x^2 = u$

$u^2 - 20u + 64 = 0 \Rightarrow u_{1,2} = 10 \pm \sqrt{100 - 64} = 10 \pm 6$

$u_1 = 16 \quad \vee \quad u_2 = 4$

Rück: Sub

$x^2 = 16 \quad \vee \quad x^2 = 4$

$x_{2,3} = \pm 4$   $\vee$   $x_{4,5} = \pm \sqrt{4} = \pm 2$

Nullstellen =  $\{-4; -2; 0; 2; +4\}$

b)  $x^5 - 17x^3 + 16x = 0$

$x \cdot (x^4 - 17x^2 + 16) = 0 \Rightarrow$

$x_1 = 0$   $\vee x^4 - 17x^2 + 16 = 0 \quad | \text{Sub. } x^2 = u$

$u^2 - 17u + 16 = 0 \Rightarrow u_{1,2} = \frac{17}{2} \pm \sqrt{\left(\frac{17}{2}\right)^2 - 16}$

$u_1 = \frac{17}{2} + \frac{15}{2} = 16 \quad \vee \quad u_2 = \frac{2}{2} = 1$

Rück Sub

$x^2 = 16 \Rightarrow \underline{x_{2,3} = \pm 4} \quad \vee \quad x^2 = 1 \Rightarrow \underline{x_{4,5} = \pm 1}$

Nullstellen:  $\{-4; -1; 0; 1; 4\}$

c)  $x^6 + 3x^4 - 54x^2 = 0 \Rightarrow x^2 \cdot (x^4 + 3x^2 - 54) = 0 \Rightarrow$

$x^2 = 0 \quad \vee \quad x^4 + 3x^2 - 54 = 0 \quad | \text{Sub } x^2 = u$

$x_1 = 0$   $u^2 + 3u - 54 = 0 \Rightarrow u_{1,2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 + 54} = -\frac{3}{2} \pm \frac{15}{2}$

$u_1 = -\frac{3}{2} + \frac{15}{2} = 6 \quad \vee \quad u_2 = -\frac{3}{2} - \frac{15}{2} = -\frac{18}{2} = -9$

Rück-Sub:  $x^2 = 6$ 

$x_{2,3} = \pm \sqrt{6}$

 $\vee x^2 = -9$   
keine weitere Lösung

Nullstellen =  $\{-\sqrt{6}; 0; +\sqrt{6}\}$