S 32 9a

$$g(x) = x^2 \;;\quad h(x) = x^3 \;,\quad \underline{f(x) = g(x)\cdot h(x)} = x^2\cdot x^3$$

$$g'(x) = 2x \;;\quad h'(x) = 3x^2 \;,$$

$$f(x) = x^2\cdot x^3 = x^5 \Rightarrow \underline{f'(x)} = 5x^4 \neq \underline{g'(x)\cdot h'(x)} = 2x\cdot 3x^2 = 6x^3$$

b) $g(x) = x^2 \;;\quad h(x) = x^3 \;;\quad x \neq 0$

$g'(x) = 2x \qquad h'(x) = 3x^2$

$$f(x) = \frac{g(x)}{h(x)} = \frac{x^2}{x^3} = \frac{1}{x} \Rightarrow \underline{f'(x)} = -\frac{1}{x^2} \neq \underline{\frac{g'(x)}{h'(x)}} = \frac{2x}{3x^2} = \frac{2}{3}\cdot\frac{1}{x}$$

aus $f(x) = g(x)\cdot h(x) \not\Rightarrow f'(x) = g'(x)\cdot h'(x)$

aus $f(x) = \dfrac{g(x)}{h(x)} \not\Rightarrow f'(x) = \dfrac{g'(x)}{h'(x)}$

Potenzregel:

$$\frac{(x_0+h)^2 - x_0^2}{h} = \frac{\cancel{x_0^2} + 2x_0h + h^2 - \cancel{x_0^2}}{h} = \frac{\cancel{h}\cdot(2x_0 + \overset{\to 0}{h})}{\cancel{h}} \quad \text{für } h\to 0$$

$$\frac{(x_0+h)^3 - x_0^3}{h} = \frac{\cancel{x_0^3} + 3x_0^2h^1 + 3x_0^1h^2 + h^3 - \cancel{x_0^3}}{h} = \frac{\cancel{h}\cdot(3x_0^2 + \overbrace{3x_0h}^{\to 0} + \overbrace{h^2}^{\to 0})}{\cancel{h}} \quad \text{für } h\to 0$$

$$\frac{(x_0+h)^4 - x_0^4}{h} = \frac{\cancel{x_0^4} + 4x_0^3h^1 + 6x_0^2\cdot h^2 + 4x_0^1h^3 + h^4 - \cancel{x_0^4}}{h}$$

für $h\to 0$

$$= \frac{\cancel{h}\cdot(4x_0^3 + \overbrace{6x_0^2\cdot h}^{\to 0} + \overbrace{4x_0h^2}^{\to 0} + \overbrace{h^3}^{\to 0})}{\cancel{h}} \Rightarrow f'(x) = 4x_0^3$$

$\vdots$

$$\underline{\underline{f(x) = x^n}} \Rightarrow m_{x_0}(h) = \frac{\cancel{x_0^n} + n\cdot x_0^{n-1}\cdot\cancel{h} + \overbrace{h^{\cancel{2}}\cdot(\text{Polynom}(x_0;h))}^{\to 0 \text{ für } h\to 0} - \cancel{x_0^n}}{\cancel{h}}$$

$$\Rightarrow \lim_{h\to 0}\left(n\cdot x_0^{n-1} + h\cdot(\text{Polynom}(x_0;h))\right) = \underline{\underline{n\cdot x_0^{n-1} = f'(x_0)}}$$