S 32 Nr 8

a) $k(x) = x^2 \qquad m_{x_0}(h) = \frac{(x_0+h)^2 - x_0^2}{h} \Rightarrow \lim_{h\to 0} m_{x_0}(h) = 2x_0$

$h(x) = x^3 \qquad m_{x_0}(h) = \frac{(x_0+h)^3 - x_0^2}{h} \Rightarrow \lim_{h\to 0} m_{0}(h) = 3x_0^2$

$$f(x) = k(x) + h(x) \Rightarrow m_{x_0}(h) = \frac{(x_0+h)^2 + (x_0+h)^3 - [x_0^2 + x_0^3]}{h}$$

$$m_{x_0}(h) = \frac{(x_0+h)^2 - x_0^2}{h} + \frac{(x_0+h)^3 - x_0^3}{h}$$

$$\Rightarrow \lim_{h\to 0} m_{x_0}(h) = \lim_{h\to 0} \frac{(x_0+h)^2 - x_0}{h} + \lim_{h\to 0} \frac{(x_0+h)^3 - x_0^3}{h}$$

$$\Rightarrow \underline{\underline{f'(x_0) = k'(x_0) + h'(x_0)}}$$

b) Es sei $k(x)$ und $h(x)$ ableitbar in $\mathbb{D}_f = \mathbb{D}_k \cap \mathbb{D}_h$

$$\Rightarrow \lim_{h\to 0} \frac{k(x_0+h) - k(x_0)}{h} = k'(x_0) \text{ existiert}$$

und $\lim_{h\to 0} \frac{h(x_0+h) - h(x_0)}{h} = h'(x_0)$ existiert

$$\Rightarrow f(x) = k(x) + h(x)$$

$$m_{x_0}(h) = \frac{f(x_0+h) - f(x_0)}{h} = \frac{k(x_0+h) + h(x_0+h) - [k(x_0) + h(x_0)]}{h}$$

$$m_{x_0}(h) = \underbrace{\frac{k(x_0+h) - k(x_0)}{h}}_{\text{Fur } h\to 0 \Rightarrow k'(x_0)} + \underbrace{\frac{h(x_0+h) - h(x_0)}{h}}_{\text{Fur } h\to 0 \Rightarrow h'(x_0)}$$

$$\Rightarrow \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h\to 0} \frac{k(x_0+h) - k(x_0)}{h} + \lim_{h\to 0} \frac{h(x_0+h) - h(x_0)}{h}$$

$$f'(x_0) = h'(x_0) + h'(x_0)$$