S 32 Nr 7

a) $f(x) = 2x^2$

$$m_{x_0}(h) = \frac{2(x_0+h)^2 - 2x_0^2}{h} = 2 \cdot \frac{x_0^2 + 2x_0h + h^2 - x_0^2}{h}$$

$$m_{x_0}(h) = 2 \cdot \frac{x_0 \cdot (2x_0 + h^2)}{x_0} = 2 \cdot (2x_0 + h^2)$$

$$\lim_{h \to 0} 2 \cdot (2x_0 + h^2) = 2 \cdot \boxed{2x_0} = f'(x_0)$$

$g'(x_0)$

$g(x) = x^2$

$$m_{x_0}(h) = \frac{(x_0+h)^2 - x_0^2}{h} = \frac{x_0^2 + 2x_0h + h^2 - x_0^2}{h} = 2x_0 + h$$

$$\lim_{h \to 0} 2x_0 + h = \boxed{2x_0} = g'(x_0)$$

$$\Rightarrow f(x) = 2 \cdot g(x) \Rightarrow f'(x) = 2 \cdot g'(x)$$

b) $g(x) = \frac{1}{x}$

$$m_{x_0}(h) = \frac{\frac{1}{x_0+h} - \frac{1}{x_0}}{h} = \frac{\frac{x_0 - (x_0+h)}{(x_0+h) \cdot x_0}}{h} = \frac{x_0 - x_0 - h}{(x_0+h) \cdot x_0 \cdot h} = \frac{-1}{(x_0+h) \cdot x_0}$$

$$\lim_{h \to 0} \frac{-1}{(x_0+h) \cdot x_0} = \frac{-1}{x_0 \cdot x_0} = \boxed{-\frac{1}{x_0^2}} = g'(x_0)$$

$f(x) = -5 \cdot \frac{1}{x}$

$$m_{x_0}(h) = \frac{-5 \frac{1}{x_0+h} - \left[-5 \cdot \frac{1}{x_0}\right]}{h} = -5 \cdot \frac{\frac{1}{x_0+h} - \frac{1}{x_0}}{h}$$

Dieser Grenzwert für $h \to 0$ existiert s.o.

$$\lim_{h \to 0} -5 \cdot \frac{-1}{(x_0+h) \cdot x_0} = -5 \cdot \boxed{\frac{-1}{x_0^2}} = -5 \cdot g'(x_0)$$

c) Es sei $g(x)$ differenzierbar $\Rightarrow \lim_{h \to 0} m_{x_0}(h) = \lim_{h \to 0} \frac{g(x_0+h) - g(x_0)}{h} = g'(x_0)$

$$f(x) = a \cdot g(x) \Rightarrow m_{x_0}(h) = \frac{a \cdot g(x_0+h) - a \cdot g(x_0)}{h} = a \cdot \underbrace{\frac{g(x_0+h) - g(x_0)}{h}}$$

$\lim_{h \to 0}$ existiert und ist $g'(x_0)$

$$\Rightarrow f'(x_0) = a \cdot g'(x_0)$$