S 23 Nr. 3

a) $f(x) = x^2 \,;\; x_0 = 3$

$$m_{x_0}(h) = \frac{(x_0+h)^2 - x_0^2}{h} = \frac{x_0^2 + 2x_0h + h^2 - x_0^2}{h} = 2x_0 + h$$

$$\lim_{h\to 0} m_{x_0}(h) = \lim_{h\to 0}(2x_0 + h) = \underline{\underline{2x_0 = f'(x_0)}}$$

$\Rightarrow \underline{\underline{f'(3) = 2\cdot 3 = 6}}$

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b) analog zu Aufgabe Nr.2

$f'(x_0) = -4\cdot x_0$ für $x_0 = 3 \Rightarrow \underline{\underline{f'(3) = -4\cdot 3 = -12}}$

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c) analog zu Aufgabe Nr. 2

$f'(x_0) = 4\cdot x_0$ für $x_0 = 3 \Rightarrow \underline{\underline{f'(3) = 4\cdot 3 = 12}}$

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d) $x_0 = 1 \Rightarrow \underline{\underline{f'(1) = 4\,1 = 4}}$

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e) $f(x) = \frac{1}{x}$, $x_0 = -1$

auf den Hauptnenner gebracht

$$m_{x_0}(h) = \frac{\frac{1}{x_0+h} - \frac{1}{x_0}}{h} = \frac{\frac{x_0 - (x_0+h)}{(x_0+h)\cdot x_0}}{h} = \frac{x_0 - (x_0+h)}{(x_0+h)\cdot x_0\cdot h}$$

$$m_{x_0}(h) = \frac{-h}{(x_0+h)\cdot x_0\cdot h} = \frac{-1}{x_0^2 + h\,x_0} \Rightarrow \lim_{h\to 0}\frac{-1}{x_0^2 + hx_0} = \underline{\underline{\frac{-1}{x_0^2} = f'(x_0)}}$$

für $x_0 = -1 \Rightarrow \underline{\underline{f'(-1) = \frac{-1}{(-1)^2} = -1}}$

f) für $x_0 = 4 \Rightarrow \underline{\underline{f'(4) = \frac{-1}{4^2} = -\frac{1}{16}}}$

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g) $f'(x_0) = +\frac{3}{x_0^2}$; $f'(4) = +\frac{3}{4^2} = +\frac{3}{16}$

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h) $f'(x_0) = -1$, $\underline{\underline{f'(3) = -1}}$

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i) $f'(x_0) = 0$; $\underline{\underline{f'(7) = 0}}$