S 23 Nr 1

a) $f(x) = x^2 \; ; \; x_0 = 3$

$$m_3(h) = \frac{(3+h)^2 - 3^2}{h} = \frac{9 + 6h + h^2 - 9}{h} = \frac{6h + h^2}{h} = \frac{h \cdot (6+h)}{h} = 6 + h$$

Der Term $6+h$ ist unproblematisch wenn $h \to 0$ strebt

$\Rightarrow m_3(h)$ hat einen Grenzwert für $h \to 0$

$$\lim_{h \to 0} m_3(h) = \lim_{h \to 0} \frac{(3+h)^2 - 3^2}{h} = \lim_{h \to 0} (6+h) = \underline{\underline{6 = f'(3)}}$$

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b) $f(x) = 2x^2 \; ; \; x_0 = 1$

$$m_1(h) = \frac{2 \cdot (1+h)^2 - \{2 \cdot 1^2\}}{h} = \frac{2 \cdot (1 + 2h + h^2) - 2}{h}$$

$$m_1(h) = \frac{\cancel{2} + 4h + 2h^2 - \cancel{2}}{h} = \frac{4h + 2h^2}{h} = \frac{h\,(4 + 2h)}{h} = 4 + 2h$$

$$\Rightarrow \lim_{h \to 0} (m_1(h)) = \lim_{h \to 0} (4 + 2h) = \underline{\underline{4 = f'(1)}}$$

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c) $f(x) = -x^2 \; ; \; x_0 = 2$

$$m_2(h) = \frac{\overbrace{-(2+h)^2}^{f(x_0+h)} - \overbrace{\{-2^2\}}^{f(x_0)}}{h} = \frac{-(4 + 4h + h^2) + 4}{h} = \frac{-4h - h^2}{h}$$

$$m_2(h) = \frac{h \cdot (-4 - h)}{h} = -4 - h$$

$$\Rightarrow \lim_{h \to 0} m_2(h) = \lim_{h \to 0} (-4 - h) = \underline{\underline{-4 = f'(2)}}$$