S 111 Nr 7

a) $f_1(x) = x^3 - 2x + 3 = 3 \mid -3 \Rightarrow x^3 - 2x = 0 = x(x^2 - 2) \Rightarrow x_1 = 0$

$x_2 = -\sqrt{2}$

$x_3 = +\sqrt{2}$

$\underline{\underline{f_1(0) = f_1(-\sqrt{2}) = f_1(+\sqrt{2}) = 3}}$

$f_2(x) = x^3 + x - 7 = 3 \mid -3 \Rightarrow x^3 + x - 10 = 0 \Rightarrow$ mit probieren $\underline{\underline{x_1 = 2}}$ gefunden

Polynomdivision

$$
\begin{array}{l}
x^3 + x - 10 = (x-2)\cdot(x^2 + 2x + 5) \\
-(x^3 - 2x^2) \\
\quad 2x^2 + x - 10 \\
\quad -(2x^2 - 4x) \\
\qquad 5x - 10 \\
\qquad \underline{5x - 10} \\
\qquad\quad 0
\end{array}
$$

Wenn weitere Lösungen vorhanden sein sollen muss $x^2 + 2x + 5 = 0$ sein

$x_{2,3} = -1 \pm \sqrt{\underbrace{1^2 - 5}_{<0}}$ $\Rightarrow$ keine weitere Lösung

$\underline{\underline{f_2(2) = 3}}$

$f_3(x) = x^4 - 6x^2 + 3 = 3 \mid -3 \Rightarrow x^4 - 6x^2 = 0 = x^2(x^2 - 6)$

$\Rightarrow x_1 = 0 \lor x_2 = -\sqrt{6} \lor x_3 = +\sqrt{6}$

$\underline{\underline{f_3(0) = f_3(-\sqrt{6}) = f_3(+\sqrt{6}) = 3}}$

b) $f(x) = (x-4)^3 + 5$ oder $f(x) = (x-4)(x+80)(x-15) + 5$

$f(4) = (4-4)^3 + 5$

$f(4) = 0 + 5 = 5$

$f(4) = (4-4)(4+80)(4-15) + 5$

$f(4) = \underbrace{0 \cdot 84 \cdot (-11)}_{=0} + 5 = 0 + 5 = 5$